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Significance of exchange area adjustment in zone modelling

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Abstract—Direct exchange areas, a basic ingredient of the zone method, must obey certain constraints, which represent conservation of radiant energy. This is, however, never possible in practice because of errors in the estimation and usage of the areas. Recently, Larsen and Howell suggested an optimization procedure for adjusting these areas to conform to the constraints. This procedure is applied, in the present work, to two practical problems for evaluating the significance of the area adjustment in the modelling of thermal radiation using the zone method. The study reveals that the type of weighting factor selected in the method of Lagrange multipliers used by Larsen and Howell for adjustment of the areas is immaterial since the smoothing itself is found to have no appreciable effect on the zone method predictions.

1. INTRODUCTION

THE PIONEERING efforts by Hottel in the field of thermal radiation led to the development of the zone method for the analysis of radiative heat exchange in practical fired heater enclosures. Although it has a basic drawback that it can be readily applied only to rectangular geometries [1] and cylindrical geometries [2], it enjoyed considerable popularity among the researchers in the field ever since it was proposed. Because of its capability to make accurate predictions, it is considered even today as a standard for testing other radiation models.

The basis of the zone method is the evaluation of direct exchange areas between all possible pairs of zones, which the enclosure is made up of. These exchange areas signify the amount of radiation originating from one zone which arrives directly at the other zone without interference from any other zone. There are three kinds of exchange areas in an enclosure consisting of an absorbing/emitting medium: surface-to-surface, surface-to-gas and gas-to-gas exchange areas. Each of these can be expressed as a multiple integral, the integrand of which is a function of the relative orientation in the enclosure of the zone pair under consideration and the absorption coefficient separating the zone pair. Hottel and Cohen [1] evaluated these multiple integrals for the rectangular geometry and presented the exchange area data in the form of several plots. Similar data were compiled by Erkku [3] for the cylindrical geometry.

By definition, the direct exchange area between a pair of zones represents flux per unit emissive power from one zone to the other [2]. It follows, then, that the sum of all the exchange areas representing flux from any one zone to each of the others in the enclosure, including itself, must equal the energy originating from that zone. For a surface zone j, this means

$$\sum_i s_i s_j + \sum_i g_i s_j = A_j \tag{1}$$

and for a volume zone k

$$\sum_i s_i g_k + \sum_i g_i g_k = 4KV_k. \tag{2}$$

The above relations are called the conservation constraints for the direct exchange areas, which must be satisfied by a complete and valid set of exchange areas computed in any zoned enclosure. Normally this never happens because of a host of errors like round-off, truncation, interpolation, etc. in the individual areas. If the exchange areas are calculated using Monte-Carlo methods, as Richter and Bauersfeld [4] and Vercammen and Froment [5] have done, inaccuracies could also creep in due to statistical errors inherent in the usage of the method.

Recently, Larsen and Howell [6] have suggested a least squares smoothing procedure using Lagrange multipliers for adjusting the exchange areas in order to satisfy the conservation constraints. The technique attempts to adjust each individual exchange area in proportion to its original size. Although their work speaks of a rational method for the smoothing process, it does not address itself to the basic issue whether it is necessary at all, from the practical view point, to adjust the areas. Besides, it raises some interesting questions regarding the sanctity of the adjustment procedure, choice of weighting factors, etc. It is with a view to exploring some of these aspects that the present investigation is undertaken. The effect of smoothing the exchange areas on the zone method predictions is studied with reference to a specific situation, where exact solutions of radiative heat fluxes are available. A second case study is also made to find out the role of the weighting factors on the adjustment process in a problem involving a large set of exchange areas.

NOMENCLATURE

- A area of a surface zone [m^2]
- E black emissive power [$W\ m^{-2}$]
- gg direct exchange area between two gas zones [m^2]
- H objective function
- K absorption coefficient of the medium [m^{-1}]
- M total number of zones in the enclosure
- q radiative heat fluxes predicted by the zone method with raw exchange areas [$kW\ m^{-2}$]
- q' radiative heat fluxes predicted by the zone method with smoothed exchange areas [$kW\ m^{-2}$]
- q^* dimensionless net radiative flux, q/E_{ad}
- sg, gs direct exchange area between a surface zone and a gas zone [m^2]
- ss direct exchange area between two surface zones [m^2]
- T temperature [K]
- T^* dimensionless temperature, T/T_{ad}
- V volume of a gas zone [m^3]
- w weighting factors used in the objective function
- $[X]$ matrix of all direct exchange areas [m^2]
- x raw exchange area [m^2]
- x' adjusted exchange area [m^2].

Greek symbols

- δ area deficit of any conservation constraint [m^2]
- λ Lagrange multipliers [m^2].

Subscripts

- ad adiabatic
- g furnace gases.

2. EXCHANGE AREA ADJUSTMENT

The raw exchange areas are adjusted using the least squares smoothing based on the method of Lagrange multipliers, as suggested by Larsen and Howell [6].

The initial estimates of direct exchange areas x_{ij} may be represented in the form of a symmetric matrix

$$X = \begin{bmatrix} [ss] & [sg] \\ [sg]^T & [gg] \end{bmatrix}. \tag{3}$$

An objective function H, defined as

$$H = \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{1}{2w_{ij}} (x_{ij} - x'_{ij})^2 \tag{4}$$

is minimized subject to the conservation constraints (equations (1) and (2)) using Lagrange multipliers (λ) to obtain the revised estimates x'_{ij}

$$x'_{ij} = x_{ij} + w_{ij}(\lambda_i + \lambda_j). \tag{5}$$

The multipliers can be obtained by solving a system of simultaneous linear equations, details of which may be found in Larsen and Howell [6]. For the special case of $w_{ij} = 1$, the unknowns can be analytically obtained from

$$\lambda_i = \frac{\delta_i}{M} - \frac{1}{2M^2} \sum_{j=1}^{M} \delta_j \tag{6}$$

in terms of the deficits δ of each row/column.

3. CASE STUDIES

3.1. *Case study I*

The first case investigated refers to a cubic fired heater employed by De Marco and Lockwood [7] for calculating radiation in furnaces using a flux model. The enclosure has a side of 10 m and it is specified that the volumetric energy release rate inside is uniform at 20 kW m^{-3}. Temperatures and emissivities are prescribed on some walls and the other walls are treated as adiabatic. The enclosure has four zones in each direction and thus there are 96 surface zones and 64 gas zones, making a total of 160 zones. The exchange area data of Tucker [8] has been used for calculating the direct exchange areas for all possible zone pair combinations. These areas are further processed using standard methods [2] to evaluate total exchange areas, which enable the calculation of heat fluxes on the walls.

Two separate calculations have been made, one, with and the other, without adjusting the exchange areas. The results of the calculations are presented in Table 1 for two absorption coefficients $K = 0.05$ and 0.2. These refer to the net heat fluxes on a specific wall, for which Monte-Carlo solutions, termed 'exact' by De Marco and Lockwood, are available for comparison purposes. The wall has 16 surface zones each being a square of 2.5 m side.

It is apparent from Table 1 that the zone method predictions of the radiative fluxes agree well with the exact values. It is more of interest, however, that the smoothing process does not seem to have any significant effect on the fluxes calculated. The relative percentage variation between the fluxes calculated with the raw and the refined exchange areas has a maximum of 0.17 for $K = 0.05$ and 1.39 for $K = 0.2$.

3.2. *Case study II*

For getting a reconfirmation of the above results in a system involving a much larger set of exchange areas and also for studying the role of the weighting factors

Table 1. Zone method predictions of radiative heat fluxes [kW m^{-2}]

	$K = 0.05$				$K = 0.20$		
Exact (7)	q	q'	$\lvert q-q'\rvert/q\times 100$	Exact (7)	q	q'	$\lvert q-q'\rvert/q\times 100$
59.1	64.61	64.67	0.09	51.4	55.08	55.81	1.325
62.7	66.7	66.88	0.16	53.4	55.56	56.33	1.386
58.6	63.21	63.32	0.17	50.8	51.18	51.86	1.329
53.9	53.87	53.92	0.09	45.8	41.35	41.83	1.160
74.9	81.35	81.39	0.05	72.9	74.37	75.35	1.318
81.3	83.71	83.71	0.00	75.2	74.61	75.62	1.353
74.1	78.67	78.69	0.03	69.9	68.31	69.20	1.303
62.0	66.32	66.41	0.14	55.6	54.54	55.19	1.192
88.8	90.69	90.70	0.01	79.8	85.04	86.03	1.164
88.7	93.55	93.46	0.10	80.6	85.51	86.54	1.205
83.2	88.00	87.95	0.06	77.0	78.58	79.49	1.158
74.5	73.65	73.73	0.11	61.5	62.55	63.22	1.071
94.3	95.33	95.27	0.06	86.8	90.95	91.62	0.737
98.5	97.81	97.74	0.07	95.1	91.56	92.29	0.797
85.8	93.37	93.29	0.09	87.7	86.13	86.76	0.731
77.4	79.97	79.93	0.06	67.2	71.25	71.70	0.632

Table 2. Deviations from conservation criteria

	Absolute		Relative (%)	
	Max	Mean	Max	Mean
Case I ($K = 0.2$)	0.1207	0.04107	0.9748	0.3445
Case II	0.1826E−02	0.564E−03	0.6867	0.2753

Table 3. Differences in exchange areas before and after smoothing

	Absolute		Relative (%)	
	Max	Mean	Max	Mean
Case I ($K = 0.2$)	0.09741	0.4840E−03	4.146	0.04717
Case II	0.7047E−03	0.2105E−05	1.971	0.02004

in the smoothing process, the zone method has been applied to a reformer furnace.

Details regarding the type of the fired heater and the zoning system adapted can be found in ref. [9]. The enclosure is made up of a total number of 292 zones, of which 220 are surface zones and 72 are gas zones. The direct exchange area data are again compiled using correlations of Tucker [8] and the adjustment is made using the method of Lagrange multipliers, as earlier. Profiles of the furnace gas temperature and net radiative heat flux on the sink surface are presented in Figs. 1 and 2 for both the cases of raw and adjusted exchange areas. These confirm the earlier conclusion that the smoothing of the exchange areas has a marginal effect on the zone method predictions.

As Larsen and Howell [6] point out, it is essential that the raw exchange area data should be reasonably accurate. It has been shown that the smoothing of the exchange areas does not cause large-scale changes in the zone method predictions. Under these circumstances, one may, with confidence, assume that the weighting factors used in the smoothing process will not significantly alter the adjusted areas to cause

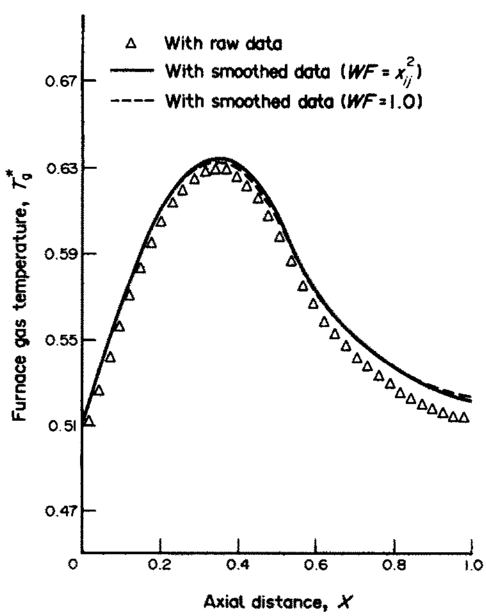

FIG. 1. Effect of exchange area adjustment on gas temperature.

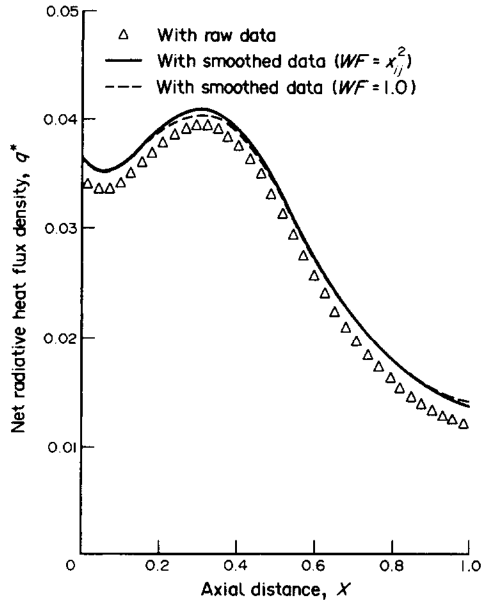

FIG. 2. Effect of exchange area adjustment on radiative flux.

any serious changes in the zone method predictions. This has indeed proved to be the case. The calculations have been performed so far with a weighting factor x_{ij}^2 for all the area elements. A separate calculation has been made adjusting the exchange areas with a uniform weighting factor of x_{ij} and meeting the conservation constraints to the same degree as with the x_{ij}^2 case. It has been rather interesting to note that the profiles of temperature and heat fluxes coincided with those obtained for the x_{ij}^2 case. The calculations have been repeated with a weighting factor of 1.0 for all the elements and the results are shown in Figs. 1 and 2 through dotted lines. It is apparent that the new profiles differ from the earlier ones very marginally. One drawback with using unity weighting factors, it was found, is that zero-valued exchange areas (i.e. elements on the same plane) get adjusted to negative values, as was also observed by Larsen and Howell [6]. As negative exchange areas are not physically meaningful, the calculations have been repeated with unity weighting factors with the proviso that the old values be retained in place of any negative areas. The new profiles calculated overlapped those obtained for the x_{ij}^2 case. This reinforces, thus, the original contention that the weighting factors should not affect the zone method predictions to any extent whatsoever.

From the foregoing discussion, one could draw several inferences. The raw area data, one normally starts with, is accurate enough to yield reliable predictions. Since true values for the individual elements are not known (this is particularly true with large systems), there is no standard against which the smoothing can be checked and hence any adjustment done is entirely arbitrary. When the values are adjusted proportionate to their magnitude, it is possible some areas move away from their true values. The conservation constraints may be satisfied but it is possible that there will be more inaccurate areas after the smoothing than before. As all weighting factors are seen to lead to the same results, one may as well use unity weighting factors, if one is interested in adjusting the raw data to meet the constraints. The reason for this is that the evaluation of the Lagrange multipliers becomes analytical in this case and hence the optimization is performed in much less time than with other weighting factors. The best way to force the direct exchange areas to meet the conservation constraints lies, however, in the development of accurate estimation procedures for the exchange areas themselves, like the one given by Siddall [10].

4. CONCLUSIONS

It has been established that the type of weighting factors employed in the method of Lagrange multipliers used for smoothing the raw exchange areas has no effect on the final outcome, i.e. the zone method predictions. This being the case, the use of unity weighting factors for all the elements is recommended, as the solution method becomes analytical and is many times faster than with other weighting factors.

Adjustment of direct exchange areas for satisfying the conservation constraints does not result in any marked improvement in the zone method predictions. This is because the original estimates of the exchange areas are very near their true values. As the exact nature of the deficiencies in the raw data are often not known, any adjustment carried out will be arbitrary. Mere satisfaction of the conservation constraints does not guarantee that more areas are nearer their true values than before the adjustment. A better solution to this problem appears to lie in a more accurate estimation of the exchange areas through a minimization of the errors involved.

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SENSIBILITE DE L'AJUSTEMENT DES SURFACES D'ECHANGE DANS LE MODELE DES ZONES

Résumé—Les surfaces d'échanges directs, un élément fondamental de la méthode des zones, doivent obéir à certaines contraintes qui représentent la conservation de l'énergie radiative. Ceci n'est pas possible en pratique à cause des erreurs dans l'estimation et l'utilisation des surfaces. Récemment Larsen et Howell ont suggéré une procédure d'optimisation pour ajuster ces surfaces aux contraintes. Cette procédure est appliquée ici à deux problèmes pratiques pour évaluer l'ajustement des surfaces dans la modélisation du rayonnement thermique en utilisant la méthode des zones. L'étude révèle que le type de facteur de pondération, sélectionné dans la méthode des multiplicateurs de Lagrange, utilisé par Larsen et Howell pour l'ajustement des surfaces est sans intérêt car le lissage lui-même n'a pas d'effet sensible sur les prédictions de la méthode des zones.

BEDEUTUNG DER ANORDNUNG DER STRAHLUNGSAUSTAUSCHENDEN FLÄCHE BEIM ZONENMODELL

Zusammenfassung—Die Flächen mit direktem Strahlungsaustausch müssen bestimmten aufgezwungenen Bedingungen gehorchen, nämlich der Erhaltung der Strahlungsenergie. Dies ist jedoch in der Praxis niemals möglich, wegen der Fehler bei der Berechnung und der Verwendung der Flächen. Kürzlich haben Larsen und Howell ein Optimierungsverfahren vorgeschlagen, um diese Flächen so anzupassen, daß die Bedingungen erfüllt sind. Dieses Verfahren wird in der vorliegenden Arbeit auf zwei praktische Probleme angewandt, wobei die Bedeutung der Flächenanpassung bei der Modellierung des Strahlungswärmeaustausches nach dem Zonenverfahren gezeigt wird. Ein Ergebnis der vorliegenden Untersuchung ist, daß der Typ des Gewichtungsfaktors unwesentlich ist, welcher beim Verfahren der Lagrange-Multiplikatoren von Larsen und Howell zur Flächenanpassung gewählt wird. Der Grund dafür ist, daß die Anpassung selbst keinen wesentlichen Einfluß auf die Berechnungen mit dem Zonenverfahren hat.

РОЛЬ ПОДБОРА ОБЛАСТЕЙ ОБМЕНА ПРИ ЗОНАЛЬНОМ МОДЕЛИРОВАНИИ

Аннотация—Области прямого обмена, являющиеся основным компонентом зонального метода, должны удовлетворять некоторым условиям сохранения энергии излучения. Однако это невозможно реализовать на практике из-за погрешности в оценке и использовании этих областей. Ларсен и Хауэлл предложили методику оптимизации их подбора при некоторых ограничениях. В данной работе предложенная методка применяется для решения двух практических задач оценки роли подбора областей обмена при моделировании теплового излучения зональным методом. Выявлено, что вид весового коэффициента, выбранного в методе множителей Лагранжа, который использовался Ларсеном и Хауэллом для подбора областей обмена, является несущественным, так как оказалось, что сглаживание не оказывает значительного влияния на расчеты с использованием зонального метода.